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POPULATION MODELS OF KOLMOGOROV-FISHER TYPE WITH DOUBLE NONLINEARITY AND NONLINEAR CROSS – DIFFUSION

MUKHAMEDIYEVA DILDORA KABILOVNA

Junior Researcher, Development Center of Software Products under TUIT, Tashkent, Toshkent Shahri, Uzbekistan

ABSTRACT

In this paper considered the model of two competing populations with double nonlinear diffusion and three types of functional dependencies. The first dependence corresponds to Malthusian type of demographic processes, the second - Ferkhulst (logistics population), and the third the population of the “Allee” type. The common element of this kind of description is the presence of a linear source. And in the descriptions of populations of Ferkhulst and Allee type also presents nonlinear sinks. Proposed appropriate initial approximation for quickly convergence iterative process. Numerical experiments are carried out with visualization for different values of the system parameters. Modeling of the processes of growth of dissipative structures in reaction-diffusion (RD) systems contributes to the development of the theoretical ideas about the colonial organization of populations.

KEYWORDS: Biological Population, Nonlinear System of Differential Equations, Initial Approximation, Numerical, Iterative Process, Self-Similar Solutions

1. INTRODUCTION

Population model of Kolmogorov Fisher type with nonlinear cross-diffusion considered in [1-12].

Let us explain what we mean by the term "cross-diffusion" (or cross-diffusion). Consider the following system of two equations in one-dimensional case:

$$\begin{aligned}\frac{\partial u_1}{\partial t} &= f(u_1,u_2) + D_1\frac{\partial^2 u_1}{\partial x^2} + h_1\frac{\partial}{\partial x}\left(Q_1(u_1,u_2)\frac{\partial u_2}{\partial x}\right),\\ \frac{\partial u_2}{\partial t} &= g(u_1,u_2) + D_2\frac{\partial^2 u_2}{\partial x^2} + h_2\frac{\partial}{\partial x}\left(Q_2(u_1,u_2)\frac{\partial u_1}{\partial x}\right).\end{aligned} \tag{1}$$

Cross-diffusion means that spatial move one object, described one of the variables is due to the diffusion of another object, described by another variable. At the population level simplest example is a parasite (the first object, located within the "host" (the second object) moves through the diffusion of the owner. The term "self-diffusion" (diffuse, direct diffusion, ordinary diffusion moves individuals at the expense of the diffusion flow from areas of high concentration, particularly in the area of low concentration. The term "cross-diffusion" means moving/thread of one species/ substances due to the presence of the gradient other individuals/ substances. The value of a cross-diffusion coefficient can be positive, negative or equal to zero. The positive coefficient of cross-diffuse indicates that the movement of individuals takes place in the direction of low concentrations of other species occurs in the direction of the high

concentration of other types of individuals/ substances. In the nature of the system with cross-diffusion quite common and play a significant role especially in biophysical and biomedical systems.

Equation (1) is a generalization of the simple diffusion model for the logistic model of population growth [13-16] of Malthus type ($f_1(u_1,u_2)=u_1$, $f_1(u_1,u_2)=u_2$, $f_2(u_1,u_2)=u_1$, $f_2(u_1,u_2)=u_2$), Ferkhulst type ($f_1(u_1,u_2)=u_1(1-u_2)$, $f_1(u_1,u_2)=u_2(1-u_1)$, $f_2(u_1,u_2)=u_1(1-u_2)$, $f_2(u_1,u_2)=u_2(1-u_1)$), and Allee type ($f_1(u_1,u_2)=u_1(1-u_2^{\beta_1})$, $f_1(u_1,u_2)=u_2(1-u_1^{\beta_2})$, $f_2(u_1,u_2)=u_1(1-u_2^{\beta_1})$, $f_2(u_1,u_2)=u_2(1-u_1^{\beta_2})$, $\beta_1>1,\ \beta_2>1$) for the case of double nonlinear diffusion. In the case when $\beta_1\geq 1,\ \beta_2\geq 1$, it can be regarded as the equation of nonlinear filtration, thermal conductivity at simultaneous influence of the source and the absorption capacity of which is equal to respectively $u_1,-u_2^{\beta_1}$, $u_2,-u_1^{\beta_2}$, exposed by convective transfer rate $\frac{\partial u_1}{\partial x}$ and $\frac{\partial u_2}{\partial x}$.

Consider the three-dimensional analogue of the Volterra-Lotka competition with nonlinear power dependence of the diffusion coefficient on population density. In the simplest case of Volterra competitive interactions between populations can construct numerically, and in some cases analytically inhomogeneous in space solutions [19].

2. LOCALIZATION OF THE WAVE SOLUTIONS OF REACTION-DIFFUSION SYSTEMS WITH DOUBLE NONLINEARITY

Consider in the domain Q={(t,x): 0< t < ∞, x∈ R^2} parabolic system of two quasilinear equations of reaction-diffusion tasks of biological populations of Kolmogorov-Fisher type

$$\begin{cases} \frac{\partial u_1}{\partial t}=\frac{\partial}{\partial x}\left(D_1 u_2^{m_1-1}\left|\frac{\partial u_1}{\partial x}\right|^{p-2}\frac{\partial u_1}{\partial x}\right)+l(t)\frac{\partial u_1}{\partial x}+k_1(t)u_1\left(1-u_2^{\beta_1}\right), \\ \frac{\partial u_2}{\partial t}=\frac{\partial}{\partial x}\left(D_2 u_1^{m_2-1}\left|\frac{\partial u_2}{\partial x}\right|^{p-2}\frac{\partial u_2}{\partial x}\right)+l(t)\frac{\partial u_2}{\partial x}+k_2(t)u_2\left(1-u_1^{\beta_2}\right), \end{cases} \tag{2}$$

$$u_1\big|_{t=0}=u_{10}(x),\ u_2\big|_{t=0}=u_{20}(x),$$

which describes the process of biological populations in nonlinear two-component environment, and its diffusion coefficient is equal to $D_1 u_2^{m_1-1}\left|\frac{\partial u_1}{\partial x}\right|^{p-2}$, $D_2 u_1^{m_2-1}\left|\frac{\partial u_2}{\partial x}\right|^{p-2}$ и_and convective transport with speed $l(t)$, where $m_1, m_2, p, \beta_1, \beta_2$ - positive real numbers, $u_1=u_1(t,x)\geq 0$, $u_2=u_2(t,x)\geq 0$ - required solutions.

The Cauchy problem and boundary problems for the system (1) in the univariate and multivariate cases investigated by many authors [15-21].

Purpose of this work is to find estimates for solutions and emerging with a free boundary that gives the chance to choose the appropriate initial approximation [15] for each value of the numeric parameters.

It is known that nonlinear equations have wave solutions in the form of diffusion waves. Under the wave meant self-similar solution of equation (2) in the form

$$u(t,x)=f(\xi),\ \xi=ct\pm x,$$

where the constant C is the wave speed.

Let's build self-similar system of equations (2) - more simple for research systems of equations. Construct a self-similar system of equations by nonlinear splitting method [15].

Replacing in (2)

$$u_1(t,x)=e^{-\int_0^t k_1(\zeta)d\zeta}v_1(\tau(t),\eta),\ \eta=x-\int_0^t l(\zeta)d\zeta,$$

$$u_2(t,x)=e^{-\int_0^t k_2(\zeta)d\zeta}v_2(\tau(t),\eta),\ \eta=x-\int_0^t l(\zeta)d\zeta,$$

lead (2) to the form:

$$\begin{cases}\dfrac{\partial v_1}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial\eta}\right|^{p-2}\dfrac{\partial v_1}{\partial\eta}\right)-k_1(t)e^{[(2-p)k_1+(\beta_1-m_1+1)k_2]t}v_1 v_2^{\beta_1},\\ \dfrac{\partial v_2}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial\eta}\right|^{p-2}\dfrac{\partial v_2}{\partial\eta}\right)-k_2(t)e^{[(\beta_2-m_2+1)k_1+(2-p)k_2]t}v_1^{\beta_2}v_2,\end{cases} \tag{3}$$

$$v_1\big|_{t=0}=v_{10}(\eta),\ v_2\big|_{t=0}=v_{20}(\eta).$$

If $k_1(p-(m_1+1))=k_2(p-(m_2+1))$, then by selecting $\tau(t)=\dfrac{e^{[(m_1-1)k_2+(p-2)k_1]t}}{(m_1-1)k_2+(p-2)k_1}=\dfrac{e^{[(m_2-1)k_1+(p-2)k_2]t}}{(m_2-1)k_1+(p-2)k_2}$, we get the following system of equations:

$$\begin{cases}\dfrac{\partial v_1}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial\eta}\right|^{p-2}\dfrac{\partial v_1}{\partial\eta}\right)-a_1(t)\tau^{b_1}v_1 v_2^{\beta_1},\\ \dfrac{\partial v_2}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial\eta}\right|^{p-2}\dfrac{\partial v_2}{\partial\eta}\right)-a_2(t)\tau^{b_2}v_1^{\beta_2}v_2,\end{cases} \tag{4}$$

where $a_1=k_1\left((p-2)k_1+(m_1-1)k_2\right)^{b_1}$, $b_1=\dfrac{(2-p)k_1+(\beta_1-m_1+1)k_2}{(p-2)k_1+(m_1-1)k_2}$,

$$a_2=k_2\left((m_2-1)k_1+(p-2)k_2\right)^{b_2},\ b_2=\dfrac{(\beta_2-m_2+1)k_1+(2-p)k_2}{(m_2-1)k_1+(p-2)k_2}.$$

If $b_i=0$, and $a_i(t)=const$, $i=1,2$, then the system has the form:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1} \left| \dfrac{\partial v_1}{\partial \eta} \right|^{p-2} \dfrac{\partial v_1}{\partial \eta} \right) - a_1 v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1} \left| \dfrac{\partial v_2}{\partial \eta} \right|^{p-2} \dfrac{\partial v_2}{\partial \eta} \right) - a_2 v_1^{\beta_2} v_2. \end{cases}$$

With the purpose of obtaining self-similar system for the system of equations (4) firstly we find solution of the system of ordinary differential equations

$$\begin{cases} \dfrac{d\overline{v}_1}{d\tau} = -a_1 \overline{v}_1 \overline{v}_2^{\beta_1}, \\ \dfrac{d\overline{v}_2}{d\tau} = -a_2 \overline{v}_1^{\beta_2} \overline{v}_2, \end{cases}$$

in the form

$$\overline{v}_1(\tau) = c_1(\tau + T_0)^{-\gamma_1}, \ \overline{v}_2(\tau) = c_2(\tau + T_0)^{-\gamma_2}, \ T_0 > 0,$$

where

$$c_1 = 1, \ \gamma_1 = \frac{1}{\beta_2}, \ c_2 = 1, \ \gamma_2 = \frac{1}{\beta_1}.$$

And then the solution of system (3)-(4) is sought in the form

$$\begin{aligned} v_1(t,\eta) &= \overline{v}_1(t) w_1(\tau,\eta), \\ v_2(t,\eta) &= \overline{v}_2(t) w_2(\tau,\eta), \end{aligned} \tag{5}$$

and $\tau = \tau(t)$ is selected as

$$\tau_1(\tau) = \int_0^\tau \overline{v}_1^{(p-2)}(t) \overline{v}_2^{(m_1-1)}(t)dt = \begin{cases} \dfrac{1}{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}(T+\tau)^{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}, & if \ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)] \neq 0, \\ \ln(T+\tau), & if \ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)] = 0, \\ (T+\tau), & if \ p=2 \ and \ m_1 = 1, \end{cases}$$

If $\gamma_1(p-2) + \gamma_2(m_1 - 1) = \gamma_2(p-2) + \gamma_1(m_2 - 1)$.

Then for $w_i(\tau, x), \ i = 1,2$ we obtain a system of equations

$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 w_2^{m_1-1} \left| \dfrac{\partial w_1}{\partial \eta} \right|^{p-2} \dfrac{\partial w_1}{\partial \eta} \right) + \psi_1 (w_1 w_2^{\beta_1} - w_1) \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 w_1^{m_2-1} \left| \dfrac{\partial w_2}{\partial \eta} \right|^{p-2} \dfrac{\partial w_2}{\partial \eta} \right) + \psi_2 (w_2 w_1^{\beta_2} - w_2) \end{cases}, \tag{6}$$

where

$$\psi_1 = \begin{cases} \dfrac{1}{(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])\tau}, & if \;\; 1-[\gamma_1(p-2)+\gamma_2(m_1-1)>0, \\ \gamma_1 \tilde{n}_1^{-(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])}, & if \;\; 1-[\gamma_1(p-2)+\gamma_2(m_1-1)=0, \end{cases} \tag{7}$$

$$\psi_2 = \begin{cases} \dfrac{1}{(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])\tau}, & if \;\; 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]>0, \\ \gamma_2 \tilde{n}_1^{-(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])}, & if \;\; 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]=0. \end{cases}$$

Representation of system (2) in the form (5) suggests that, when $\tau \to \infty$, $\psi_i \to 0$ and

$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 w_2^{m_1-1} \left| \dfrac{\partial w_1}{\partial \eta} \right|^{p-2} \dfrac{\partial w_1}{\partial \eta} \right), \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 w_1^{m_2-1} \left| \dfrac{\partial w_2}{\partial \eta} \right|^{p-2} \dfrac{\partial w_2}{\partial \eta} \right). \end{cases} \tag{8}$$

Therefore, the solution of system (1) with condition (5) tends to the solution of the system (8).

If $1-[\gamma_1(p-2)+\gamma_2(m_1-1)=0$, wave solution of system (6) has the form

$$w_i(\tau(t),\eta) = y_i(\xi), \; \xi = c\tau \pm \eta, \; i = 1,2,$$

where c - velocity of the wave, and the fact that the equation for $w_i(\tau, x)$ without the younger members always has a self-similar solution if $1-[\gamma_1(p-2)+\gamma_2(m_1-1) \neq 0$ we get the system

$$\begin{cases} \dfrac{d}{d\xi}\left(y_2^{m_1-1} \left| \dfrac{dy_1}{d\xi} \right|^{p-2} \dfrac{dy_1}{d\xi}\right) + c\dfrac{dy_1}{d\xi} + \psi_1 (y_1 - y_1 \, y_2^{\beta_1}) = 0, \\ \dfrac{d}{d\xi}\left(y_1^{m_2-1} \left| \dfrac{dy_2}{d\xi} \right|^{p-2} \dfrac{dy_2}{d\xi}\right) + c\dfrac{dy_2}{d\xi} + \psi_2 (y_2 - y_2 \, y_1^{\beta_2}) = 0. \end{cases}$$

After integration (8) we get the system of nonlinear differential equations of the first order

$$\begin{cases} y_2^{m_1-1}\left|\dfrac{dy_1}{d\xi}\right|^{p-2}\dfrac{dy_1}{d\xi}+cy_1=0, \\ y_1^{m_2-1}\left|\dfrac{dy_2}{d\xi}\right|^{p-2}\dfrac{dy_2}{d\xi}+cy_2=0. \end{cases} \quad (9)$$

System (9) has an approximate solution of the form

$$\bar{y}_1 = A(a-\xi)^{\gamma_1}\,,\ \bar{y}_2 = B(a-\xi)^{\gamma_2}\,,$$

where

$$\gamma_1 = \frac{(p-1)(p-(m_1+1))}{(p-2)^2-(m_1-1)(m_2-1)}\,,$$

$$\gamma_2 = \frac{(p-1)(p-(m_2+1))}{(p-2)^2-(m_1-1)(m_2-1)}\,.$$

And the coefficients A and B are determined from the solution of a system of nonlinear algebraic equations

$$(\gamma_1)^{p-1}A^{p-1}B^{m_1-1}=c\,,$$

$$(\gamma_2)^{p-1}A^{m_2-1}B^{p-1}=c\,.$$

Then taking into account expressions

$$u_1(t,x)=e^{-\int_0^t k_1(\zeta)d\zeta}\,v_1(\tau(t),\eta)\,,$$

$$u_2(t,x)=e^{-\int_0^t k_2(\zeta)d\zeta}\,v_2(\tau(t),\eta)$$

we have

$$u_1(t,x)=Ae^{-\int_0^t k_1(\zeta)d\zeta}\,(c\tau(t)-\xi)_+^{\gamma_1}\,,$$

$$u_2(t,x)=Be^{-\int_0^t k_2(\zeta)d\zeta}\,(c\tau(t)-\xi)_+^{\gamma_2}\,,\ c>0\,.$$

Due to the fact that

$$[b\tau(t)-\int_0^t l(\eta)d\eta - x]=0\,,$$

if

$$x \geq [b\tau(t) - \int_0^t l(\eta)d\eta - x] < 0\,,\ \forall t > 0\,,$$

then

$$u_1(t,x) \equiv 0\,,\ u_2(t,x) \equiv 0\,, x \geq [b\tau(t) - \int_0^t l(\eta)d\eta - x] < 0\,,\ \forall t > 0\,.$$

Therefore, the condition of localization of solutions of (2) are the conditions

$$\int_0^e l(y)dy < 0\,,\ \tau(t) < \infty \text{ for } \forall t > 0\,. \tag{10}$$

Condition (10) is the condition for the emergence of a new effect - the localization of the wave solutions (2). If the condition (10) unfulfilled, then there is the phenomenon of the finite speed of propagation of a perturbation, i.e. $u_i(t,x) \equiv 0$ at $|x| \geq b(t)$, $\tau(t) = \int_0^t e - {}^{(m_1+p-3)\int_0^\zeta k_1(y)dy} d\zeta$, and front goes arbitrarily far away, with increasing time since $\tau(t) \to \infty$ at $t \to \infty$.

Below we consider the properties of solutions of systems of parabolic equations with cross diffusion.

3. POPULATION MODELS OF KOLMOGOROV-FISHER TYPE WITH NONLINEAR CROSS-DIFFUSION

Let's consider in the domain Q={(t,x): 0< t < ∞, x∈R^2} parabolic system of two quasilinear equations with nonlinear cross – diffusion

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left(D_1 u_2^{m_1-1}\left|\dfrac{\partial u_2}{\partial x}\right|^{p-2} \dfrac{\partial u_2}{\partial x}\right) + l(t)\dfrac{\partial u_1}{\partial x} + k_1(t)u_1\left(1 - u_2^{\beta_1}\right) \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left(D_2 u_1^{m_2-1}\left|\dfrac{\partial u_1}{\partial x}\right|^{p-2} \dfrac{\partial u_1}{\partial x}\right) + l(t)\dfrac{\partial u_2}{\partial x} + k_2(t)u_2\left(1 - u_1^{\beta_2}\right) \end{cases} \tag{11}$$

$$u_1\big|_{t=0} = u_{10}(x)\,,\ u_2\big|_{t=0} = u_{20}(x)\,,$$

which describes the process of biological populations in nonlinear two-component environment, which diffusion coefficient is equal $D_1 u_2^{m_1-1}\left|\dfrac{\partial u_2}{\partial x}\right|^{p-2}$, $D_2 u_1^{m_2-1}\left|\dfrac{\partial u_1}{\partial x}\right|^{p-2}$ and convective transfer with speeds $l(t)$, where $m_1, m_2, p, \beta_1, \beta_2$ - positive real numbers, $u_1 = u_1(t,x) \geq 0$, $u_2 = u_2(t,x) \geq 0$ - search solutions.

Let's build self-similar system of equations (11) - more simple for research of systems of equations.

Construct a self-similar system of equations by nonlinear splitting method [15].

Replacement in (11)

$$u_1(t,x)=e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),\eta),\ \eta = x-\int_0^t l(\zeta)d\zeta,$$

$$u_2(t,x)=e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),\eta),\ \eta = x-\int_0^t l(\zeta)d\zeta,$$

lead (11) to the form:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta}\right)-k_1(t)e^{[(\beta_1-m_1-p+2)k_2-k_1]t}v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta}\right)-k_2(t)e^{[(\beta_2-m_2-p+2)k_1-k_2]t}v_1^{\beta_2} v_2, \end{cases} \tag{12}$$

$$v_1\big|_{t=0}=v_{10}(\eta),\ v_2\big|_{t=0}=v_{20}(\eta).$$

By choosing $\tau(t)=\dfrac{e^{[(m_1+p-2)k_2+k_1]t}}{(m_1+p-2)k_2+k_1}=\dfrac{e^{[(m_2+p-2)k_1+k_2]t}}{(m_2+p-2)k_1+k_2}$, we obtain the following system of equations:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta}\right)-a_1(t)\tau^{b_1}v_1 v_2^{\beta_1} \\ \dfrac{\partial v_2}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta}\right)-a_2(t)\tau^{b_2}v_1^{\beta_2} v_2 \end{cases} \tag{13}$$

where $a_1=k_1\left((m_1+p-2)k_2-k_1\right)^{b_1}$, $b_1=\dfrac{(\beta_1-m_1-p+2)k_2-k_1}{(m_1+p-2)k_2-k_1}$,

$$a_2=k_2\left((m_2+p-2)k_1-k_2\right)^{b_2},\ b_2=\frac{(\beta_2-m_2-p+2)k_1-k_2}{(m_2+p-2)k_1-k_2}.$$

If $b_i=0$, and $a_i(t)=const, i=1,2$, then the system has the form:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta}\right)-a_1 v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta}\right)-a_2 v_1^{\beta_2} v_2. \end{cases}$$

Below we describe one way of obtaining self-similar system for the system (13). It consists in the following. We first find the solution of ordinary differential equations

$$\begin{cases} \dfrac{d\bar{v}_1}{d\tau} = -a_1 \bar{v}_1 \bar{v}_2^{\beta_1}, \\ \dfrac{d\bar{v}_2}{d\tau} = -a_2 \bar{v}_1^{\beta_2} \bar{v}_2, \end{cases}$$

in the form

$$\bar{v}_1(\tau) = c_1(\tau + T_0)^{-\gamma_1}, \ \bar{v}_2(\tau) = c_2(\tau + T_0)^{-\gamma_2}, \ T_0 > 0,$$

where

$$c_1 = 1, \ \gamma_1 = \frac{1}{\beta_2}, \ c_2 = 1, \ \gamma_2 = \frac{1}{\beta_1}.$$

And then the solution of (12) - (13) searched in the form

$$\begin{aligned} v_1(t,\eta) &= \bar{v}_1(t) w_1(\tau,\eta), \\ v_2(t,\eta) &= \bar{v}_2(t) w_2(\tau,\eta), \end{aligned} \tag{14}$$

and $\tau = \tau(t)$ is selected as

$$\tau_1(\tau) = \int_0^{\tau} \bar{v}_1^{(p-2)}(t) \bar{v}_2^{(m_1-1)}(t)dt = \begin{cases} \dfrac{1}{1-[\gamma_1(m_1+p-2)+\gamma_2]}(T+\tau)^{1-[\gamma_1(m_1+p-2)+\gamma_2]}, & if \ 1-[\gamma_1(m_1+p-2)+\gamma_2] \neq 0, \\ \ln(T+\tau), & if \ 1-[\gamma_1(m_1+p-2)+\gamma_2] = 0, \\ (T+\tau), & if \ m_1 + p = 2, \end{cases}$$

if $\gamma_1(m_1 + p - 1) = \gamma_2(m_2 + p - 1)$.

Then for $w_i(\tau, x), \ i = 1,2$ we obtain a system of equations

$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 w_2^{m_1-1} \left| \dfrac{\partial w_2}{\partial \eta} \right|^{p-2} \dfrac{\partial w_2}{\partial \eta} \right) + \psi_1 (w_1 w_2^{\beta_1} - w_1) \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 w_1^{m_2-1} \left| \dfrac{\partial w_1}{\partial \eta} \right|^{p-2} \dfrac{\partial w_1}{\partial \eta} \right) + \psi_2 (w_2 w_1^{\beta_2} - w_2) \end{cases}, \tag{15}$$

where

$$\psi_1 = \begin{cases} \dfrac{\gamma_1}{1-[\gamma_1(m_1+p-2)+\gamma_2]\tau}, & if \ 1-[\gamma_1(m_1+p-2)+\gamma_2 > 0, \\ \gamma_1 \tilde{n}_1^{-(\gamma_1(m_1+p-2)+\gamma_2)}, & if \ 1-[\gamma_1(m_1+p-2)+\gamma_2 = 0. \end{cases} \tag{16}$$

$$\psi_2 = \begin{cases} \dfrac{\gamma_2}{1-[\gamma_2(m_2+p-2)+\gamma_1]\tau}, & if\ \ 1-[\gamma_2(m_2+p-2)+\gamma_1>0, \\ \gamma_1\tilde{n}_1^{-(\gamma_2(m_2+p-2)+\gamma_1)}, & if\ \ 1-[\gamma_2(m_2+p-2)+\gamma_1=0. \end{cases}$$

Presentation system (11) as (14) suggests that when $\tau \to \infty$, $\psi_i \to 0$ and

$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 w_2^{m_1-1}\left|\dfrac{\partial w_2}{\partial \eta}\right|^{p-2}\dfrac{\partial w_2}{\partial \eta}\right), \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 w_1^{m_2-1}\left|\dfrac{\partial w_1}{\partial \eta}\right|^{p-2}\dfrac{\partial w_1}{\partial \eta}\right). \end{cases} \tag{17}$$

Therefore, the solution of system (11) with condition (14) tends to the solution of the system (17).

Wave solution of the system (17) has the form

$w_i(\tau(t),\eta) = y_i(\xi)$, $\xi = c\tau \pm \eta$, $i = 1,2$,

where c - speed of the wave, and the fact that the equation for $w_i(\tau, x)$ without the younger members always has a self-similar solution if $1-[\gamma_1(m_1+p-2)+\gamma_2 \neq 0,$ we obtain the system

$$\begin{cases} \dfrac{d}{d\xi}(y_2^{m_1-1}\left|\dfrac{dy_2}{d\xi}\right|^{p-2}\dfrac{dy_2}{d\xi}) + c\dfrac{dy_1}{d\xi} + \psi_1(y_1 - y_1\ y_2^{\beta_1}) = 0, \\ \dfrac{d}{d\xi}(y_1^{m_2-1}\left|\dfrac{dy_1}{d\xi}\right|^{p-2}\dfrac{dy_1}{d\xi}) + c\dfrac{dy_2}{d\xi} + \psi_2(y_2 - y_2\ y_1^{\beta_2}) = 0. \end{cases}$$

After integration (17) we get the system of nonlinear differential equations of the first order

$$\begin{cases} y_2^{m_1-1}\left|\dfrac{dy_2}{d\xi}\right|^{p-2}\dfrac{dy_2}{d\xi} + cy_1 = 0, \\ y_1^{m_2-1}\left|\dfrac{dy_1}{d\xi}\right|^{p-2}\dfrac{dy_1}{d\xi} + cy_2 = 0. \end{cases} \tag{18}$$

The system (18) has an approximate solution of the form

$\bar{y}_1 = A(a-\xi)^{\gamma_1}$, $\bar{y}_2 = B(a-\xi)^{\gamma_2}$,

where A and B find from the system of algebraic equations $(\gamma_2)^{p-1}A^{-1}B^{m_1+p-2} = c$,

$(\gamma_1)^{p-1}A^{m_2+p-2}B^{-1} = c$,

and $\gamma_1 = \dfrac{(p-1)(1-(m_1+p))}{1-(2-(m_1+p))(2-(m_2+p))}$,

$$\gamma_2 = \frac{(p-1)(1-(m_2+p))}{1-(2-(m_1+p))(2-(m_2+p))}.$$

Then taking into account expressions

$$u_1(t,x) = e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),\eta),$$

$$u_2(t,x) = e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),\eta)$$

we have

$$u_1(t,x) = Ae^{-\int_0^t k_1(\zeta)d\zeta} (c\tau(t)-\xi)_+^{\gamma_1},$$

$$u_2(t,x) = Be^{-\int_0^t k_2(\zeta)d\zeta} (c\tau(t)-\xi)_+^{\gamma_2}, \; c>0.$$

The study of qualitative properties of the system (2) allowed to perform numerical experiment based on the values included in the system of numerical parameters. For this purpose, as the initial approximation was used asymptotic solutions. For the numerical solving of the task for the linearization of system (2) has been used linearization methods of Newton and Picard. To build self-similar system of equations of biological populations used the method of nonlinear splitting [16,19].

4. NUMERICAL EXPERIMENT

For the numerical solving of the task (2) construct a uniform grid

$$\omega_h = \{x_i = ih, \;\; h>0, \; i=0,1,...,n, \;\; hn=l\},$$

and the temporary net

$$\omega_{h_1} = \{t_j = jh_1, \qquad h_1>0, \; j=0,1,...,n, \;\; \tau m = T\}.$$

Replace the task (2) by implicit difference scheme and receive differential task with the error $O(h^2+h_1)$.

As you know, the main problem for the numerical solution of nonlinear task is the appropriate choice of the initial approximation and the method of linearization of the system (2).

Consider the function:

$$v_{10}(t,x) = \nu_1(t)\cdot(a-\xi)_+^{\gamma_1},$$

$$v_{20}(t,x) = \nu_2(t)\cdot(a-\xi)_+^{\gamma_2},$$

where $\nu_1(t) = e^{kt}\overline{\nu}_1(t)$ и $\nu_2(t) = e^{kt}\overline{\nu}_2(t)$ defined above functions,

Note $(a)_+$ means that $(a)_+ = \max(0, a)$. These functions are finite speed of propagation of perturbations [16,19]. Therefore, for the numerical solving of the task (1)-(2) $\beta_1 > \sigma_1$ as an initial approximation offered functions $v_{i0}(t,x)$, $i = 1,2$.

Established in the input language MathCad program allows visually trace the evolution of the process for different values of parameters and data.

Numerical calculations show that in the case of arbitrary values $\sigma > 0, \beta > 0$ qualitative properties of solutions are not changed. Below listed results of numerical experiments for different values of parameters (Figure 1 – Figure 3).

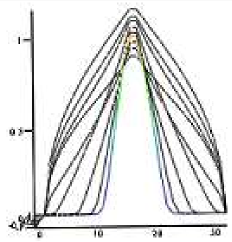
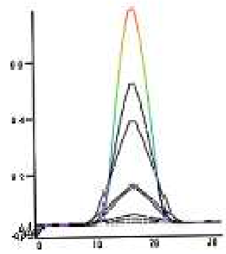
Parameter Values	Results of Numerical Experiment
$m_1 = 2.1$, $m_2 = 2.3$, $p = 3$ $k_1 = 0.1$ $k_2 = 0.25$ $eps = 10^{-3}$	
$m_1 = 2.3$, $m_2 = 2.5$, $p = 3.5$ $k_1 = 5$ $k_2 = 7$ $eps = 10^{-3}$	

Figure 1: Dynamics of Malthusian Population

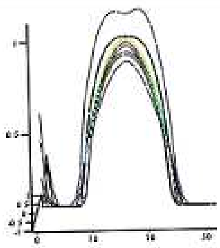
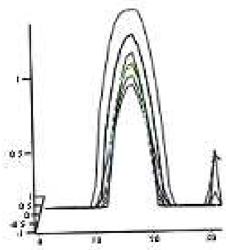
Parameter Values	Results of Numerical Experiment
$m_1 = 2.1$, $m_2 = 2.3$, $p = 3$ $\beta_1 = 1$, $k_1 = 7$ $\beta_2 = 1$, $k_2 = 9$ $eps = 10^{-3}$	
$m_1 = 2.3$, $m_2 = 2.5$, $p = 3.5$ $\beta_1 = 1$, $k_1 = 3$ $\beta_2 = 1$, $k_2 = 2$ $eps = 10^{-3}$	

Figure 2: Dynamics of Logistic Population

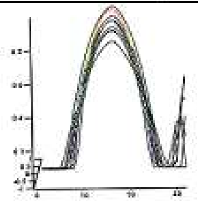
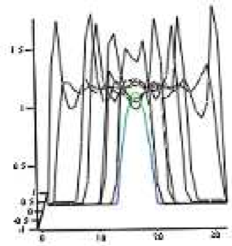
Parameter Values	Results of Numerical Experiment
$m_1 = 2.1$, $m_2 = 2.3$, $p = 3$ $\beta_1 = 2$, $k_1 = 0.2$ $\beta_2 = 2$, $k_2 = 0.9$ $eps = 10^{-3}$	
$m_1 = 2.3$, $m_2 = 2.5$, $p = 3.5$ $\beta_1 = 5$, $k_1 = 2$ $\beta_2 = 7$, $k_2 = 9$ $eps = 10^{-3}$	

Figure 3: Dynamics of Allee Effect

5. CONCLUSIONS

In the future will be explored theoretical properties of systems with cross-diffusion, which will make a significant contribution to the study of nonlinear systems of differential equations.

REFERENCES

1. Ахромеева ТСи др. *Нестационарные структуры и диффузионный хаос* (М.: Наука, 1992)
2. de1-Casti11o-Negrete D, Carreras B A Phys. Plasmas 9 118 (2002)
3. del-Castillo-Negrete D, Сагге В A, Lynch V Physica D 168-169 45 (2002)
4. Jorne J L Theor. Biol. 55 529 (1975)
5. A1mirantis Y, Papageorgiou S L Theor. Biol. 151 289 (1991)
6. Иваницкий Г Р, Медвинский А Б, Цыганов М А *УФН*161 (4) 13 (1991)
7. Wu Y, Zhao X Physica D 200 325 (2005)
8. Kuznetsov Yu A et a1. L Math. Biol. 32 219 (1994)
9. Кузнецов Ю А и др. "Кросс-диффузионная модель динамики границы леса", в сб. *Проблемы экологического мониторинга и моделирования экосистем* Т. 26 (СПб.: Гидрометиздат, 1996) с. 213
10. Burridge R, KnopoffL Вм//. Seismol. Soc. Am. 57 341 (1967)
11. Cartwright J HE, Hernandez-Garcia E, Piro O Phys. Pev. Lett. 79 527(1997)
12. Stuart A M /MA L Math. *Appl.* Med. Biol. 10 149 (1993)
13. Murray J.D. Mathematical Biology. I. An Introduction (Third Edition). – N.Y., Berlin, Heidelberg: Springer Verlag, 2001. – 551 p.,

14. M. Aripov (1997). «Approximate Self-similar Approach tu Solve Quasilinear Parabolic Equation» Experimentation, Modeling and Computation in Flow Turbulence and Combustion vol. 2. p. 19- 26.
15. Арипов М. Метод эталонных уравнений для решения нелинейных краевых задач Ташкент, Фан, 1988, 137 с.
16. Белотелов Н.В., Лобанов А.И.Популяционные модели с нелинейной диффузией. // Математическое моделирование. –М.; 1997, №12, стр. 43-56.
17. В. Вольтерра. Математическая теория борьбы за существование -М.: Наука, 1976, 288 с.
18. Гаузе Г.Ф. О процессах уничтожения одного вида другим в популяциях инфузорий // Зоологический журнал,1934, т.13, №1.
19. Aripov M., Muhammadiev J. Asymptotic behaviour of automodel solutions for one system of quasilinear equations of parabolic type. Buletin Stiintific-Universitatea din Pitesti, Seria Matematica si Informatica. N 3. 1999. pg. 19-40
20. Aripov M.M. Muhamediyeva D.K. To the numerical modeling of self-similar solutions of reaction-diffusion system of the one task of biological population of Kolmogorov-Fisher type. International Journal of Engineering and Technology. Vol-02, Iss-11, Nov-2013. India. 2013.
21. Арипов М.М. Мухамедиева Д.К. Подходы к решению одной задачи биологической популяции. Вопросы вычислительной и прикладной математики. -Ташкент. 2013. Вып.129. -С.22-31.